1-dimensional kinematics

Definitions:

Position, or location of an object, is denoted by *x.* Position is always given with respect to some zero point.

Change in position, i.e. *displacement*, is denoted by Δx,
and is defined as $\Delta x = x_2 - x_1$.

Velocity is a measure of how fast, and in which direction, an object is moving. It is denoted by *v*. The *average velocity* of an object is defined by the ratio of the displacement and the duration of the movement, as given by

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}$$

When the velocity of an object changes, the object is *accelerating*. The *average acceleration* is defined as the ratio of the change in velocity and the duration of the change;

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1}$$

Uniform motion; motion at constant velocity

x_0 and t_0 = 0:

When studying the motion of an object, the position of the object at the start of the motion (i.e. the start of the studying of the motion), is often denoted by x_0. The corresponding time is usually set to zero, since the time "starts" when the motion starts.

x and t: At any time t (other then t_0), the object has position x.

Using these symbols, the (average = constant) velocity is given by the expression

$$v = \frac{x - x_0}{t}$$

We can reshape this into an expression for the position *x* at time *t*.

$$x = x_0 + vt$$

Uniformly accelerated motion; motion at constant acceleration (steadily changing velocity)

When an object has constant acceleration, its velocity changes at a steady rate.

v_0 and $t_0 = 0$:

When the object starts accelerating, it has a starting velocity v_0. The corresponding time t_0 is usually set to zero.

v and t:

At some time *t*, the velocity iof the object is *v*.

We can now give an expression for the acceleration:

$$a = \frac{v - v_0}{t}$$

We can reshape this into an expression for the velocity at any time *t*:

$$v = v_0 + at$$

At constant acceleration, we can give the *average* velocity as

$$\bar{v} = \frac{v_0 + v}{2}$$

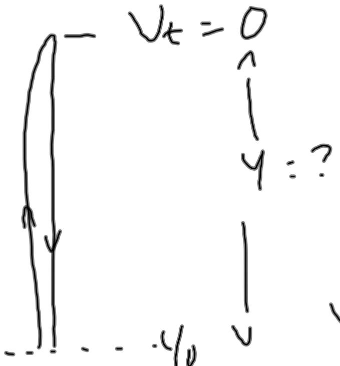
The displacement, at constant acceleration, is given by

$$\Delta x = v_0 t + \frac{1}{2} a t^2$$

We can combine the above expressions into an expression for the final velocity, using only displacement, acceleration and the initial velocity:

$$v^2 = {v_0}^2 + 2 \cdot a \cdot \Delta x$$

The derivation is left as a challenge for the reader :)



a) $v_t = 0 \iff$

$v_0 + gt = 0$

$t = -\frac{v_0}{g}$

$$y_t = v_0 \cdot t + \frac{1}{2} g t^2$$
$$= v_0 \cdot \left(-\frac{v_0}{g}\right) + \frac{1}{2} g \cdot \left(-\frac{v_0}{g}\right)^2$$
$$= -\frac{v_0^2}{g} + \frac{1}{2} \cdot g \cdot \frac{v_0^2}{g^2}$$
$$= -\frac{v_0^2}{g} + \frac{1}{2} \frac{v_0^2}{g} = -\frac{v_0^2}{2g}$$

Highest point

$$y = -\frac{v_0^2}{2g} = -\frac{(-5{,}0\,\frac{m}{s})^2}{2 \cdot 9{,}81\,\frac{m}{s^2}} \approx -1{,}3\,m$$

b) We know $y = -0{,}5\,m$:

$$y_t = v_0 \cdot t + \frac{1}{2} g t^2$$

$$\frac{1}{2} g t^2 + v_0 t - y = 0$$

$$t = \frac{-v_0 \pm \sqrt{v_0^2 - 4 \cdot \frac{1}{2} g (-y)}}{2 \cdot \frac{1}{2} g} = \frac{-v_0 \pm \sqrt{v_0^2 + 2g \cdot y}}{g}$$

$$= \frac{5{,}0\,\frac{m}{s} \pm \sqrt{25{,}0\,\frac{m^2}{s^2} + 2 \cdot 9{,}81\,\frac{m}{s^2} \cdot (-0{,}5\,m)}}{9{,}81\,\frac{m}{s^2}}$$

$t_1 \approx 0{,}906\,s \approx 0{,}91\,s$

$t_2 \approx 0{,}11\,s$

Calculate the muzzle velocity of the spring gun:

y_0 = 26,5 cm

$y_{highest}$ = 75,5 cm